The image depicts how our robot Irma3D sees itself in a mirror. The laser looking into itself creates distortions as well as changes in intensity that give the robot a single eye, complete with iris and pupil. Thus, the image is called "Self Portrait with Duckling".
The ICP Algorithm (1)

**Scan registration** Put two independent scans into one frame of reference

**Iterative Closest Point** algorithm [Besl/McKay 1992]

For prior point set $M$ (“model set”) and data set $D$

1. Select point correspondences $w_{i,j}$ in \{0,1\}
2. Minimize for rotation $R$, translation $t$

$$E(R, t) = \sum_{i=1}^{N_m} \sum_{j=1}^{N_d} w_{i,j} \| m_i - (R d_j + t) \|^2$$

3. Iterate 1. and 2.

SVD-based calculation of rotation

- works in 3 translation plus 3 rotation dimensions
  ⇒ 6D SLAM with closed loop detection and global relaxation.
The ICP Algorithm (2)

Closed form (one-step) solution for minimizing of the error function

1. Cancel the double sum:

\[
E(R, t) = \sum_{i=1}^{N_m} \sum_{j=1}^{N_d} w_{i,j} ||m_i - (Rd_j + t)||^2
\]

\[
\propto \frac{1}{N} \sum_{i=1}^{N} ||m_i - (Rd_i + t)||^2,
\]

2. Compute centroids of the matching points

\[
c_m = \frac{1}{N} \sum_{i=1}^{N} m_i, \quad c_d = \frac{1}{N} \sum_{i=1}^{N} d_j
\]

\[
M' = \{m'_i = m_i - c_m\}_{1,\ldots,N}, \quad D' = \{d'_i = d_i - c_d\}_{1,\ldots,N}.
\]

3. Rewrite the error function

\[
E(R, t) = \frac{1}{N} \sum_{i=1}^{N} \left||m'_i - Rd'_i - \underbrace{(t - c_m + Rc_d)}_{=:\hat{t}}\right||^2
\]
3. Rewrite the error function

\[
E(R, t) = \frac{1}{N} \sum_{i=1}^{N} \| m'_i - Rd'_i - (t - c_m + Rc_d) \|^2
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} \| m'_i - Rd'_i \|^2 - \frac{2}{N} \tilde{t} \cdot \sum_{i=1}^{N} (m'_i - Rd'_i) + \frac{1}{N} \sum_{i=1}^{N} \| \tilde{t} \|^2.
\]

Minimize only the first term! (The second is zero and the third has a minimum for \( \tilde{t} = 0 \)).

\[
E(R, t) = \sum_{i=1}^{N} \| m'_i - Rd'_i \|^2.
\]

Arun, Huang und Blostein suggest a solution based on the singular value decomposition.

The ICP Algorithm (4)

**Theorem:** Given a 3 x 3 correlation matrix

\[
H = \sum_{i=1}^{N} m_i'^T d_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}
\]

with \( S_{xx} = \sum_{i=1}^{N} m_{ix}' d_{ix}' \), \( S_{xy} = \sum_{i=1}^{N} m_{ix}' d_{iy}' \), \( \cdots \), then the optimal solution for \( E(R, t) = \sum_{i=1}^{N} \| m_i' - Rd_i' \| ^2 \) is \( R = VU^T \) with \( H = U\Lambda V^T \) from the SVD.

**Proof:**

\[
E(R, t) = \sum_{i=1}^{N} \| m_i' - Rd_i' \| ^2.
\]

Rewrite

\[
E(R, t) = \sum_{i=1}^{N} \| m_i' \| ^2 - 2 \sum_{i=1}^{N} m_i' \cdot Rd_i' + \sum_{i=1}^{N} \| d_i' \| ^2.
\]

Rotation is length preserving, i.e., maximize the term

\[
\sum_{i=1}^{N} m_i' \cdot Rd_i' = \sum_{i=1}^{N} m_i'^T Rd_i'
\]
The ICP Algorithm (5)

Theorem: Given a 3 x 3 correlation matrix

\[ H = \sum_{i=1}^{N} m_i^T d_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix} \]

with \( S_{xx} = \sum_{i=1}^{N} m_i' d_i' x, \ S_{xy} = \sum_{i=1}^{N} m_i' d_i' y, \ldots \), then the optimal solution for \( E(R, t) = \sum_{i=1}^{N} ||m_i' - Rd_i'||^2 \) is \( R = VU^T \) with \( H = U\Lambda V^T \) from the SVD.

Proof:

\[ \sum_{i=1}^{N} m_i' \cdot Rd_i' = \sum_{i=1}^{N} m_i'^T Rd_i' \]

Rewrite using the trace of a matrix

\[ \text{Trace} \left( \sum_{i=1}^{N} Rd_i' m_i'^T \right) = \text{Trace} \left( RH \right) \]

Lemma: For all positive definite matrices \( A A^T \) and all orthonormal matrices \( B \) the following equation holds:

\[ \text{Trace} \left( A A^T \right) \geq \text{Trace} \left( B A A^T \right) \]

\( \square \)
The ICP Algorithm (6)

Theorem: Given a 3 x 3 correlation matrix

\[
H = \sum_{i=1}^{N} m_i'^T d_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}
\]

with \( S_{xx} = \sum_{i=1}^{N} m_i' x_i' d_i' x_i' \), \( S_{xy} = \sum_{i=1}^{N} m_i' x_i' d_i' y_i' \), \ldots, then the optimal solution for \( E(R, t) = \sum_{i=1}^{N} \| m_i' - R d_i' \|^2 \) is \( R = VU^T \) with \( H = U\Lambda V^T \) from the SVD.

Proof: Suppose the singular value decomposition of \( H \) is \( H = U\Lambda V^T \). \( U \) and \( V \) are orthonormal 3 x 3 and \( \Lambda \) a diagonal matrix without negative entries.

\( R = VU^T \).

\( R \) is orthonormal and \( RH = VU^T U\Lambda V^T = V\Lambda V^T \).

And using the lemma it is \( \text{Trace} (RH) \geq \text{Trace} (BRH) \).
The ICP Algorithm (7)

- Estimating the transformation can be accomplished very fast $O(n)$

- Closest point search
  - Naïve $O(n^2)$, i.e., brute force
  - K-d trees for searching in logarithmic time

Recommendation: Start with
ANN: A Library for Approximate Nearest Neighbor Searching by David M. Mount and Sunil Arya (University of Maryland)
- Easy to use
- Many different methods are available
- Quite fast

http://www.cs.umd.edu/~mount/ANN/
One has to search all buckets according to the ball-within-bounds-test. ⇒ Backtracking
NNS Search – the Critical Issue

Properties for all tested NNS libraries.

<table>
<thead>
<tr>
<th>Library</th>
<th>revision</th>
<th>Data structure</th>
<th>$k$-NN search</th>
<th>fixed radius</th>
<th>ranged search</th>
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<td>shape registration</td>
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<td>×</td>
<td></td>
</tr>
</tbody>
</table>

Matching Time in s

- actuated SICK LMS200
- Microsoft Kinect
- Riegl VZ-400

Large Scale 3D Point Cloud Processing Tutorial
Dr. Andreas Nüchter
November 25, 2013
NNS Search – the Critical Issue
The ICP Algorithm (8)

• Point reduction – another key for fast ICP algorithms
  – Start with cube surrounding the 3D point cloud
The ICP Algorithm (9)

- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud
  - Divide

- Another key issue: maximal point-to-point distance.
Registering Surfaces (1)

• Given

The main idea:
  – Pairwise matching technique
  – We want to minimize the distance between the two parts
  – We set up a variational problem
  – Minimize distance “energy” by rigid motion of one part
Registering Surfaces (2)

Problem:
- How to compute the distance
- This is simple if we know the corresponding points.
- Of course, we have in general no idea of what corresponds...

• ICP-idea: set closest point as corresponding point

• Full algorithm:
  - Compute closest point points
  - Minimize distance to these closest points by a rigid motion
  - Recompute new closest points and iterate
Registering Surfaces (3)

- Distances

![Diagram of Distances](image1)

- Closest Point Distances

![Diagram of Closest Point Distances](image2)
Registering Surfaces (4) – ICP iterations

Part A

Part B

Part A

Part B

final result
Processing Large Data Sets (1)

bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
--epsICP=0.00001 ~/dat/hannover/

We see: small matching errors accumulate
Applications of 3D Mapping with ICP

CMU 3D mapping of abandoned mines

RoboCup Rescue

(video)

(video)

(video)
6D SLAM – Global Relaxation (1)

- In SLAM loop closing is the key to build consistent maps
- Notice: Consistent vs. correct or accurate

- GraphSLAM
  - Graph Estimation
  - Graph Optimization

- Graph Estimation
  - Simple strategy: Connect poses with graph edges that are close enough
  - Simple strategy: Connect poses, they have enough point pairs (closest points)
The global Algorithm

**Scan registration** Put two independent scans into one frame of reference

**Iterative Closest Point** algorithm [Besl/Mckay 1992]

For prior point set $M$ ("model set") and data set $D$

1. Select point correspondences $w_{i,j}$ in \{0,1\}
2. Minimize for rotation $R$, translation $t$

$$E(R, t) = \sum_{i=1}^{N_m} \sum_{j=1}^{N_d} w_{i,j} \left\| m_i - (Rd_j + t) \right\|^2$$

3. Iterate 1. and 2.

Four closed form solution for the minimization works in 3 translation plus 3 rotation dimensions

$$E = \sum_{j=k}^{m} \sum_{i} |R_j m_i + t_j - (R_k d_i + t_k)|^2$$

Minimize for all rotations $R$ and translations $t$ at the same time
Parametrizations for the Rigid Body Transformations

$$E = \sum_{j \rightarrow k} \sum_{i} |R_{j}m_{i} + t_{j} - (R_{k}d_{i} + t_{k})|^{2}$$

• Helix transformation

$$v(p) = \bar{x} + x \times p$$

$$E = \sum_{j \rightarrow k} \sum_{i} (m_{i} - d_{i} + (\bar{x}_{j} + x_{j} \times m_{i}) - (\bar{x}_{k} + x_{k} \times m_{i}))^{2}$$

... solving a system of linear equations
Parametrizations for the Rigid Body Transformations

\[
E = \sum_{j \rightarrow k} \sum_{i} |R_j m_i + t_j - (R_k d_i + t_k)|^2
\]

• Small angle approximation

\[
\sin \theta \approx \theta - \frac{\theta^3}{3} + \frac{\theta^5}{5} - \cdots
\]

\[
\cos \theta \approx 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} - \cdots
\]

\[
R \approx \begin{pmatrix}
1 & -\theta_z & \theta_y \\
\theta_x \theta_z + \theta_z & 1 - \theta_x \theta_y \theta_z & -\theta_x \\
\theta_x \theta_z - \theta_y & \theta_x + \theta_y \theta_z & 1
\end{pmatrix}
\]

\[
R \approx \begin{pmatrix}
1 & -\theta_z & \theta_y \\
\theta_z & 1 & -\theta_x \\
-\theta_y & \theta_x & 1
\end{pmatrix}
\]

... solving a system of linear equations
Parametrizations for the Rigid Body Transformations

\[ E = \sum_{j \rightarrow k} \sum_{i} |R_j m_i + t_j - (R_k d_i + t_k)|^2 \]

- Explicit modeling of uncertainties
- Assumptions: The unknown error is normally distributed

\[ W = \sum_{j \rightarrow k} (\bar{E}_{j,k} - E_{j,k}')^T C_{j,k}^{-1} (\bar{E}_{j,k}' - E_{j,k}') \]
\[ = \sum_{j \rightarrow k} (\bar{E}_{j,k} - (X_j' - X_k')) C_{j,k}^{-1} (\bar{E}_{j,k}' - (X_j' - X_k')). \]
\[ E_{j,k} = \sum_{i=1}^{m} \|X_j \oplus d_i - X_k \oplus m_i\|^2 = \sum_{i=1}^{m} \|Z_i(X_j, X_k)\|^2 \]

... solving a system of linear equations
**Comparisons of the Parametrizations**

<table>
<thead>
<tr>
<th>Global ICP</th>
<th>Classical Pose GraphSLAM</th>
<th>Riegl Laser Measurement GmbH</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Gaussian noise in the „3D Point Cloud“ space</td>
<td>• Gaussian noise in the space of poses</td>
<td>(video)</td>
</tr>
<tr>
<td>• Locally optimal</td>
<td>• Locally optimal</td>
<td>(video)</td>
</tr>
<tr>
<td>• ICP-like iterations using new point correspondences</td>
<td>• Gradient descent needed</td>
<td>(video)</td>
</tr>
<tr>
<td></td>
<td>• ICP-like iterations using new point correspondences needed as well</td>
<td>(video)</td>
</tr>
</tbody>
</table>
Closed Loop Detection and Global Relaxation

3D data acquisition
Processing Large Data Sets (2)

```
bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
    --epsICP=0.00001 ~/dat/hannover/
```

We see: small matching errors accumulate

```
bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
    --epsICP=0.00001
    -D 250 -I 50 --cldist=750 -L 0 -G 1
~/dat_hannover

bin/show -s 1 -e 65 ~/dat/dat_hannover
```